Toward A Mathematical Understanding of the Malware Problem

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Importance of Understanding the Problem

Will Ohmer and Orville Wright. Wright Patterson AFB.

Inventor and great grandfather Will Ohmer used to say:

"To invent something, identify and understand the problem."

Limitations of Malware Detection

Up against fundamental limits in Computer Science:

- No computer algorithm can detect all malware. [1]
- NP hard problems (traveling salesman) help encrypt and hide the malware. [2]

[1] Fred Cohen. Computer Viruses Theory and Experiments. Computers and Security. 6(1) 22–35, Feb. 1987.

[2] Eric Filiol. Malicious Cryptology and Mathematics. Cryptography and Security in Computing. Chapter 2. Intech, March 7, 2012.

Register Machine Vulnerability

- After C, JAVA or Python program is compiled, a register machine is a computer that executes the compiled instructions.
- Register machines execute one instruction at-a-time.
- Register machine hardware uses branch instructions (jump).
- A 1 or 2 bit flip in an instruction subverts the program.
- One rogue branch instruction can jump to a malware routine.
- In some cases, malware detection code won't execute.

Flipping 2 bits in a C program instruction

```
int greater_than(int p1, int p2)
{ return (p1 > p2); }

int less_than(int p1, int p2)
{ return (p1 < p2); }

int main(int argc, char* argv[])
{
   int nums[4] = {6, 9, 7, 8};
   display_numbers(nums, 4);
   printf("\n");
   sort_pr(nums, 4, "less_than", less_than);
   sort_pr(nums, 4, "greater_than", greater_than);
   return 0;
}
```

```
~MacBook-Air:sort$  ./sort
6 9 7 8

6 7 8 9  address of instruction less_than
1 1 0 1 1 0 1 0 0 1 0 0 0 0 0 0

9 8 7 6  address of instruction greater_than
1 1 0 1 1 0 1 0 0 0 0 1 0 0 0 0
```

Motivating Stable Computation

- Typical programming languages (C, JAVA, Python) use conditional branching instructions.
- 75% to 80% of control flow instructions in register machines are conditional branch instructions. [3]
- A register machine program's purpose can be subverted because its behavior is not stable w.r.t. small changes.

[3] J. Hennessy and D. Patterson. Computer Architecture. 5th Edition, Morgan Kaufmann, 2012. Figure A.14

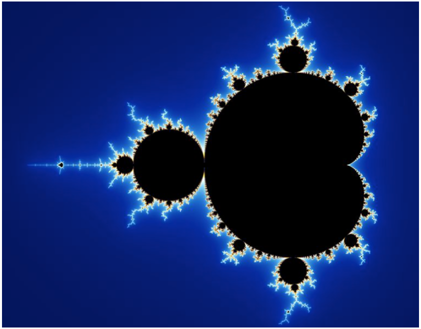
A Program is a Dynamical System

- Dynamical systems has studied stability for over 80 years. [4]
- Use mathematical tools (metric spaces, structural stability, topological conjugacy, . . .) from dynamical systems theory.
- *Small changes* to a computer program can be measured as small changes to a dynamical system.
- Each Turing machine (computer program) maps to a finite set of affine maps (2×2 matrix $+$ translation) in the x-y plane.

[4] Aleksandr Andronov and Lev Pontrjagin. Systèmes Grossiers. Dokl. Akad. Nauk., SSSR, 14, 247–251, 1937.

Mandelbrot Set is generated from a Dynamical System

Define function $f_c(z) = z^2 + c$. c is a complex number.



The Mandelbrot set is the set of all complex numbers c such that the orbit $0, \quad f_c(0), \quad f_c \circ f_c(0), \quad f_c \circ f_c \circ f_c(0), \quad \ldots$ is bounded.

ϕ Maps each Turing Instruction to a Unique Affine Map

Alphabet $A = \{\#, a, b\}$. Machine states $Q = \{q, r, s\}$.

Turing program η.

η	#	a	b
q	$(r, a, +1)$	$(h, b, +1)$	$(q, b, -1)$
r	$(q, b, -1)$	$(r, a, +1)$	$(r, b, +1)$
s	$(h, \#, +1)$	$(h, a, +1)$	$(h, b, +1)$

$$\eta(q, \#) = (r, a, +1) \quad \xrightarrow{\phi} \quad f_1(x, y) = (7x - 49,\ \tfrac{1}{7}y + 33)$$

$$\eta(r, \#) = (q, b, -1) \quad \xrightarrow{\phi} \quad f_2(x, y) = (\tfrac{1}{7}x + 16,\ 7y - 231)$$

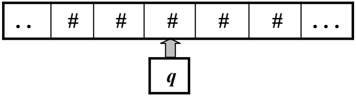
Turing Instruction Execution $\overset{\phi}{\leftrightarrow}$ Affine Map Iteration

$$\nu(h) = 0 \quad \nu(\#) = 1 \quad \nu(a) = 2 \quad \nu(b) = 3 \quad \nu(q) = 4 \quad \nu(r) = 5 \quad \nu(s) = 6$$

$$B = |Q| + |A| + 1 = 7$$

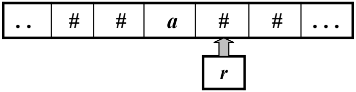
$$(q, k, T) \quad \overset{\phi}{\leftrightarrow} \quad \Big(\sum_{j=-1}^{\infty} \nu(T_{k+j+1}) B^{-j},\ B\nu(q) + \sum_{j=0}^{\infty} \nu(T_{k-j-1}) B^{-j} \Big)$$

Before machine execution starts



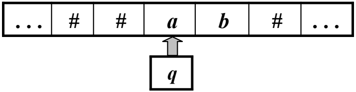
$$p_0 = (8\frac{1}{6}, 29\frac{1}{6})$$

After Instruction $\eta(q, \#) = (r, a, +1)$



$$p_1 = f_1(p_0) = (8\frac{1}{6}, 37\frac{1}{6})$$

After Instruction $\eta(r, \#) = (q, b, -1)$



$$p_2 = f_2(p_1) = (17\frac{1}{6}, 29\frac{1}{6})$$

Topological Conjugacy and Structural Stability

- A topologically conjugacy h between two dynamical systems f, g means they have equivalent dynamics.
- h maps halting configurations to fixed points in the plane.
- A halting configuration represents the result of a computation after the computer program completes its computation.
- f is structurally stable if all dynamical systems g that are close to f via some metric, then f and g are topologically conjugate.

MAIN IDEA: If Turing machine $\mathcal{M}$ is structurally stable, then SMALL CHANGES to the computer program $\mathcal{M}$ WILL NOT CHANGE WHAT $\mathcal{M}$ COMPUTES!

Can We Find Standard Computation That Is Stable?

- A Universal Turing Machine is basically a compiler or interpreter (C, JAVA, Lisp, Python, ...).
- In our paper, a Universal Turing machine (UTM) is provided and it is shown that this UTM is structurally unstable.
- This means that given a computer program P there are other programs arbitrarily close that exhibit different dynamics.
- This means some of the nearby computer programs do not perform the same computation as program P.

All Register Machine Compilers might be Unstable

- These initial results suggest that all compilers for C, Python, etc. might be structurally unstable.
- Math theory translates to computer programs executing on register machines are inherently susceptible to malware vulnerabilities.
- Caveat: instability was only shown for two metrics and one UTM encoding.

Research Questions for Conventional Machines

- Are there general mathematical conditions when a Turing machine (register machine) is structurally unstable?
- Do useful metrics exist on the space of Turing machines (computer programs)?

WITI: IF stable conditions exist, THEN WE CAN DESIGN

robust digital computer programs with our know-how.

Research Direction for Unconventional Machines

IF stable conditions DO NOT EXIST, THEN WE CAN

- BUILD machines that simultaneously execute instructions
- BUILD machines that can repair their instructions.

WITI: The program purpose can be made stable.

Computation is resistant to sabotage of instructions.