Quantum Random, Self-Modifiable Computation

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Standard Model of Computation

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With randomness and self-modification, two instances of the same initial ex-machine can execute and evolve to two distinct ex-machines. (Non-autonomous dynamical system.)

An ex-machine can make a computational mistake on a first instance of a problem and subsequently repair its program before executing on a second instance of the same problem.

Preliminaries

From $\{a\}^*$, define the set of languages $\mathfrak{L} = \bigcup_{L \subset \{a\}^*} \{L\}$.

For $f : \mathbb{N} \to \{0, 1\}$, define language $L_f = \{a^n : f(n) = 1\}$.

$$\mathfrak{L} = \bigcup_{f \in \{0,1\}^{\mathbb{N}}} \{L_f\}$$

Let μ be the Lebesgue measure on $\{0, 1\}^{\mathbb{N}}$. $\mu\big(\{0, 1\}^{\mathbb{N}}\big) = 1$.

μ induces Lebesgue measure ν on $\mathfrak{L}$ via $f \leftrightarrow L_f$. $\nu(\mathfrak{L}) = 1$.

Set Alphabet $A = \{\#, 0, 1, N, Y, a\}$. $\#$ is the blank symbol.
Set States $Q = \{0, h, n, y, t, v, w, x, 8\}$ with halting state h

$\mathfrak{Q}(x)$ Specification: 15 Initial Instructions

$(0, \#, 8, \#, 1)$
$(8, \#, x, \#, 0)$
$(y, \#, h, Y, 0)$
$(n, \#, h, N, 0)$

$(x, \#, x, 0)$
$(x, a, t, 0)$

$(x, 0, v, \#, 0, (|Q| - 1, \#, n, \#, 1))$
$(x, 1, w, \#, 0, (|Q| - 1, \#, y, \#, 1))$
$(t, 0, w, a, 0, (|Q| - 1, \#, n, \#, 1))$
$(t, 1, w, a, 0, (|Q| - 1, \#, y, \#, 1))$

$(v, \#, n, \#, 1, (|Q| - 1, a, |Q|, a, 1))$
$(w, \#, y, \#, 1, (|Q| - 1, a, |Q|, a, 1))$
$(w, a, |Q|, a, 1, (|Q| - 1, a, |Q|, a, 1))$

$(|Q| - 1, a, x, a, 0)$
$(|Q| - 1, \#, x, \#, 0)$

Main Results

$\mathfrak{Q}(x)$ computes Turing incomputable languages in $\mathfrak{L}$ with probability (Lebesgue measure) 1.

After a finite number of computational steps, $\mathfrak{Q}(x)$ uses a finite amount of computing resources.

Consider an enumeration $\mathcal{E}_a(i) = (\mathfrak{M}_i, T_i)$ of all Turing machines $\mathfrak{M}_i$ and initial tapes T_i, each containing a finite number of non-blank symbols.

There exists an evolutionary path $\mathfrak{Q}(x) \to \mathfrak{Q}_1 \to \mathfrak{Q}_2 \to \dots \to \mathfrak{Q}_m$, so at the mth stage $\mathfrak{Q}_m$ correctly determines for $0 \le i \le m$ whether $\mathfrak{M}_i$'s execution on tape T_i halts.

Key Observations

$p = \frac{1}{2}$. Our random instructions use no hidden tricks, e.g. p is Turing incomputable. [1]

In practice, $\mathfrak{Q}(x)$ will not find this evolutionary halting path, even though it is possible. (Impossible for a Turing machine.)

$\mathfrak{Q}(x)$'s dynamical behavior circumvents the contradiction in an information-theoretic proof of Turing's halting problem.[2]

The circumvention occurs because $\mathfrak{Q}(x)$'s meta instructions increase the number of states and instructions in $\mathfrak{Q}(x)$.

[1] K. de Leeuw, E.F. Moore, C. Shannon, N. Shapiro. Computability by Probabilistic Machines. Princeton University Press. 1956.

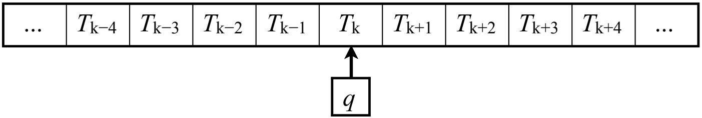
[2] C. Calude. Information and Randomness. Springer. 1994. pp. 184-185

Standard Instructions $\mathcal{S}$: (q, T_k, r, b, y)

Standard Instructions are Turing machine instructions.

$Q = \{0, 1, \ldots, n-1\} \subset \mathbb{N}$ is the set of states.

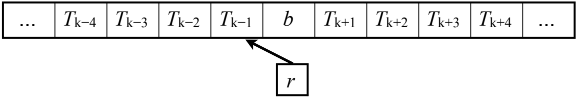
Alphabet $A = \{0, 1, \#\} \cup \{a_1, \ldots a_m\}$. $\#$ is the blank symbol.



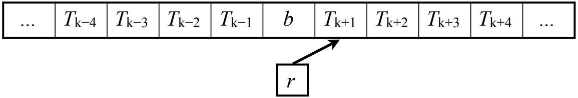
Standard Instructions $\mathcal{S}$

Reading T_k on the tape in state q, write b on tape and move to state r. Move tape head $y \in \{-1, 0, 1\}$.

$(q, T_k, r, b, -1)$



$(q, T_k, r, b, +1)$



Random Instructions $\mathcal{R}$: (q, a, r, y)

When scanning alphabet symbol a and lying in state q, random instruction (q, a, r, y) executes as follows.

Measure a quantum event that returns a random $b \in \{0, 1\}$.

On the tape, replace alphabet symbol a with random bit b. (Alphabet A always contains symbols $\{0, 1\}$.)

The *ex*-machine state moves to state r.

The *ex*-machine moves its tape head left if $y = -1$, right if $y = +1$, or does not move if $y = 0$.

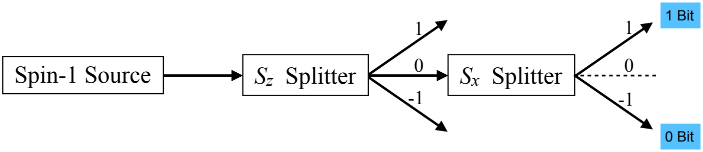
Quantum Random Axioms

Unbiased Trials: Consider bit sequence $(x_1\ x_2\ x_3\ \ldots)$ in the infinite product space $\{0,1\}^{\mathbb{N}}$. A single outcome x_i generated by quantum randomness is unbiased. The probabilities satisfy $P(x_i = 1) = P(x_i = 0) = \frac{1}{2}$.

Stochastic Independence: History has no effect on the next quantum random measurement. Each outcome x_i is independent of the history. Expressed as conditional probabilities, $P(x_i = a \mid x_1 = b_1, \ldots, x_{i-1} = b_{i-1}) = \frac{1}{2}$ for $a = 0$, $a = 1$ and for each $(b_1, b_2, \ldots, b_{i-1}) \in \{0,1\}^{i-1}$.

A Theory for a Quantum Random Number Generator

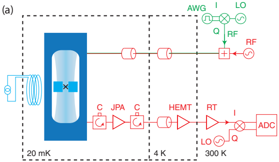
Protocol for a QRNG Based on Value Indefiniteness[3]

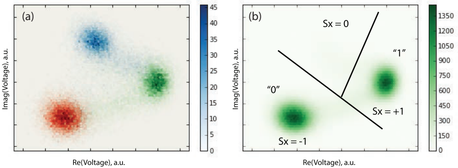


[3] Abbott, Calude, Conder, Svozil. Strong Kochen-Specker theorem and incomputability of quantum randomness. Physical Review A 86, 062109. 2012.

A Physical Realization of a QRNG

Measurement Setup and S_x Data [4]





[4] Kulikov et al. Realization of a QRNG certified with the Kochen-Specker Theorem. 2017. arXiv:1709.03687v1

Meta Instructions $\mathcal{M}$

Meta instructions can add new states.

Meta instructions can add new instructions

Meta instructions can replace instructions.

Meta instructions $\mathcal{M}$ are a subset of $\{(q, a, r, \alpha, y, J) : q \in Q$ and $r \in Q \cup \{|Q|\}$ and $a, \alpha \in A$ and instruction $J \in \mathcal{S} \cup \mathcal{R}\}$.

J is a standard or random instruction

Execution of Meta Instruction (q, a, r, α, y, J)

$\mathcal{I} = \mathcal{S} \cup \mathcal{R} \cup \mathcal{M}$.

Quintuple (q, a, r, α, y) executes as a standard instruction with one caveat:

State q may be expressed as $|Q| - c_1$ and state r may be expressed as $|Q|$ or $|Q| - c_2$, where $0 < c_1, c_2 \leq |Q|$. When (q, a, r, α, y) is executed, if q is expressed as $|Q| - c_1$, the value of q is instantiated to the current value of $|Q|$ minus c_1.

If state r is expressed as $|Q|$ or $|Q| - c_2$, the value of r instantiates to the current value of $|Q|$ or $|Q| - c_2$, respectively.

Execution of Meta Instruction (q, a, r, α, y, J)

Unique state, scanning condition: for any two distinct instructions chosen from $\mathcal{I}$ at least one of the first two coordinates must differ.

Next, instruction J modifies $\mathcal{I}$, where instruction J has one of the two forms: $J = (q, a, r, \alpha, y)$ or $J = (q, a, r, y)$.

For both forms, if $\mathcal{I} \cup \{J\}$ still satisfies the unique state, scanning symbol condition, then $\mathcal{I}$ is updated to $\mathcal{I} \cup \{J\}$.

Otherwise, there is an instruction I in $\mathcal{I}$ whose first two coordinates q, a are equal to instruction J's first two coordinates. In this case, instruction J replaces instruction I in $\mathcal{I}$. That is, $\mathcal{I}$ is updated to $\mathcal{I} \cup \{J\} - \{I\}$

Example that Executes a Meta Instruction

Consider meta instruction $(q, a_1, |Q| - 1, \alpha_1, y_1, J)$, where $J = (|Q| - 1, a_2, |Q|, \alpha_2, y_2)$.

After standard instruction $(q, a_1, |Q| - 1, \alpha_1, y_1)$ executes, this meta instruction adds a new state $|Q|$ to the states Q and adds instruction J, instantiated with the current value of $|Q|$.

Set states $Q = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Alphabet $A = \{\#, 0, 1\}$.

An initial configuration is shown below.

State	Tape
5	##11 01##

Example that Executes a Meta Instruction

Meta instruction $(5, 0, |Q| - 1, 1, 0, J)$ executes with values $q = 5$, $a_1 = 0$, $\alpha_1 = 1$, $y_1 = 0$, $a_2 = 1$, $\alpha_2 = \#$, and $y_2 = -1$.

Instruction $J = (|Q| - 1,\ 1, |Q|, \#, -1)$

Since $|Q| = 8$, instruction $(5, 0, 7, 1, 0)$ executes.
$J = (7, 1, 8, \#, -1)$ is added as a new standard instruction.

The instantiation of $|Q| = 8$ in J adds state 8. The states are updated to $Q = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

The new ex-machine configuration is shown below.

State	Tape
7	##11 11##

Example that Executes a Meta Instruction

State	Tape
7	$\#\#11\ \ 11\#\#$

Now, the ex-machine is scanning a 1 and lying in state 7, so the standard instruction $J = (7, 1, 8, \#, -1)$ executes.

Note J was just added to the instructions.

After J executes, the new configuration is shown below.

State	Tape
8	$\#\#1\ \ 1\#1\#\#$

Simple Meta Instructions

A simple meta instruction has syntax, where $0 < c_1, c_2 \leq |Q|$:

$$(q, a, |Q| - c_2, \alpha, y) \quad \text{or} \quad (q, a, |Q|, \alpha, y)$$

$$(|Q| - c_1, a, r, \alpha, y)$$

$$(|Q| - c_1, a, |Q| - c_2, \alpha, y) \quad \text{or} \quad (|Q| - c_1, a, |Q|, \alpha, y).$$

Expressions $|Q| - c_1$, $|Q| - c_2$ and $|Q|$ are instantiated to a state based on the current value of $|Q|$ when the meta instruction executes.

$\mathfrak{Q}(x)$ self-reflects with the symbols $|Q| - 1$ and $|Q|$.

Finite Initial Conditions

A *finitely bounded tape* means the tape has a finite number of non-blank symbols.

An ex-machine has *finite initial conditions* if the following 4 conditions are satisfied before the ex-machine starts executing.

1. The number of states $|Q|$ is finite.
2. The number of alphabet symbols $|A|$ is finite.
3. The number of machine instructions $|\mathcal{I}|$ is finite.
4. The tape is finitely bounded.

Evolving an Ex-machine

T_0, T_1, ... T_{i-1} are finitely bounded tapes. Ex-machine $\mathfrak{X}_0$ has finite initial conditions.

$\mathfrak{X}_0$ starts executing with tape T_0 and evolves to ex-machine $\mathfrak{X}_1$ with tape S_1.

Next, $\mathfrak{X}_1$ starts executing with tape T_1 and evolves to $\mathfrak{X}_2$ with tape S_2. This means that when ex-machine $\mathfrak{X}_1$ starts executing on tape T_1, its instructions are preserved after the halt with tape S_1.

The ex-machine evolution continues until $\mathfrak{X}_{i-1}$ starts executing with tape T_{i-1} and evolves to ex-machine $\mathfrak{X}_i$ with tape S_i.

Evolutionary Path, Ancestors and Descendants

One says that ex-machine $\mathfrak{X}_0$ with finitely bounded tapes T_0, T_1, T_2 ... T_{i-1} *evolves* to ex-machine $\mathfrak{X}_i$ after i halts.

When ex-machine $\mathfrak{X}_0$ evolves to $\mathfrak{X}_1$ and subsequently $\mathfrak{X}_1$ evolves to $\mathfrak{X}_2$ and so on up to ex-machine $\mathfrak{X}_n$, then ex-machine $\mathfrak{X}_i$ is called an *ancestor* of ex-machine $\mathfrak{X}_j$ whenever $0 \leq i < j \leq n$.

Similarly, ex-machine $\mathfrak{X}_j$ is called a *descendant* of ex-machine $\mathfrak{X}_i$ whenever $0 \leq i < j \leq n$.

The sequence of ex-machines $\mathfrak{X}_0 \to \mathfrak{X}_1 \to \ldots \to \mathfrak{X}_n \ldots$ is called an *evolutionary path*.

Languages that $\mathfrak{Q}(x)$ Evolves to Compute

Recall that $\mathfrak{L} = \bigcup_{L \subset \{a\}^*} \{L\}$. Alphabet $A = \{\#, 0, 1, N, Y, a\}$. The initial states are $Q = \{0, h, n, y, t, v, w, x, 8\}$ with halting state h

Let $\mathfrak{X}$ be an ex-machine that is a descendant of $\mathfrak{Q}(x)$. The language L in $\mathfrak{L}$ that $\mathfrak{X}$ computes is defined as follows.

A valid initial tape has the form $\# \quad \#a^n\#$. The valid initial tape $\# \quad \#\#$ represents the empty string.

After machine $\mathfrak{X}$ starts executing with initial tape $\# \quad \#a^n\#$, string a^n is in $\mathfrak{X}$'s language if $\mathfrak{X}$ halts with tape $\#a^n\# \quad Y\#$.

a^n is not in $\mathfrak{X}$'s language if $\mathfrak{X}$ halts with tape $\#a^n\# \quad N\#$.

$\mathfrak{Q}(x)$ Specification

$(0, \#, 8, \#, 1)$
$(8, \#, x, \#, 0)$
$(y, \#, h, Y, 0)$
$(n, \#, h, N, 0)$

$(x, \#, x, 0)$
$(x, a, t, 0)$

$(x, 0, v, \#, 0, (|Q| - 1, \#, n, \#, 1))$
$(x, 1, w, \#, 0, (|Q| - 1, \#, y, \#, 1))$
$(t, 0, w, a, 0, (|Q| - 1, \#, n, \#, 1))$
$(t, 1, w, a, 0, (|Q| - 1, \#, y, \#, 1))$

$(v, \#, n, \#, 1, (|Q| - 1, a, |Q|, a, 1))$
$(w, \#, y, \#, 1, (|Q| - 1, a, |Q|, a, 1))$
$(w, a, |Q|, a, 1, (|Q| - 1, a, |Q|, a, 1))$

$(|Q| - 1, a, x, a, 0)$
$(|Q| - 1, \#, x, \#, 0)$

$\mathfrak{Q}(x)$ Starts with Tape $\#\quad \#aaaa\#\#$ and State 0

STATE	TAPE	INSTRUCTION EXECUTED	NEW INSTRUCTION
8	$\#\#\quad aaaa\#\#\#$	$(0, \#, 8, \#, 1)$	
x	$\#\#\quad aaaa\#\#\#$	$(\lvert Q\rvert - 1, a, x, a, 0)$	$(8, a, x, a, 0)$
t	$\#\#\quad 1aaa\#\#\#$	$(x, a, t, 1_{qr}, 0)$	
w	$\#\#\quad aaaa\#\#\#$	$(t, 1, w, a, 0, (\lvert Q\rvert - 1, \#, y, \#, 1))$	$(8, \#, y, \#, 1)$
9	$\#\#a\quad aaa\#\#\#$	$(w, a, \lvert Q\rvert, a, 1, (\lvert Q\rvert - 1, a, \lvert Q\rvert, a, 1))$	$(8, a, 9, a, 1)$
x	$\#\#a\quad aaa\#\#\#$	$(\lvert Q\rvert - 1, a, x, a, 0)$	$(9, a, x, a, 0)$
t	$\#\#a\quad 1aa\#\#\#$	$(x, a, t, 1_{qr}, 0)$	
w	$\#\#a\quad aaa\#\#\#$	$(t, 1, w, a, 0, (\lvert Q\rvert - 1, \#, y, \#, 1))$	$(9, \#, y, \#, 1)$
10	$\#\#aa\quad aa\#\#\#$	$(w, a, \lvert Q\rvert, a, 1, (\lvert Q\rvert - 1, a, \lvert Q\rvert, a, 1))$	$(9, a, 10, a, 1)$
x	$\#\#aa\quad aa\#\#\#$	$(\lvert Q\rvert - 1, a, x, a, 0)$	$(10, a, x, a, 0)$
t	$\#\#aa\quad 0a\#\#\#$	$(x, a, t, 0_{qr}, 0)$	
w	$\#\#aa\quad aa\#\#\#$	$(t, 0, w, a, 0, (\lvert Q\rvert - 1, \#, n, \#, 1))$	$(10, \#, n, \#, 1)$
11	$\#\#aaa\quad a\#\#\#$	$(w, a, \lvert Q\rvert, a, 1, (\lvert Q\rvert - 1, a, \lvert Q\rvert, a, 1))$	$(10, a, 11, a, 1)$
x	$\#\#aaa\quad a\#\#\#$	$(\lvert Q\rvert - 1, a, x, a, 0)$	$(11, a, x, a, 0)$
t	$\#\#aaa\quad 1\#\#\#$	$(x, a, t, 1_{qr}, 0)$	
w	$\#\#aaa\quad a\#\#\#$	$(t, 1, w, a, 0, (\lvert Q\rvert - 1, \#, y, \#, 1))$	$(11, \#, y, \#, 1)$
12	$\#\#aaaa\quad \#\#\#$	$(w, a, \lvert Q\rvert, a, 1, (\lvert Q\rvert - 1, a, \lvert Q\rvert, a, 1))$	$(11, a, 12, a, 1)$
x	$\#\#aaaa\quad \#\#\#$	$(\lvert Q\rvert - 1, \#, x, \#, 0)$	$(12, \#, x, \#, 0)$
x	$\#\#aaaa\quad 0\#\#$	$(x, \#, x, 0_{qr}, 0)$	
v	$\#\#aaaa\quad \#\#\#$	$(x, 0, v, \#, 0, (\lvert Q\rvert - 1, \#, n, \#, 1))$	$(12, \#, n, \#, 1)$
n	$\#\#aaaa\#\quad \#\#$	$(v, \#, n, \#, 1, (\lvert Q\rvert - 1, a, \lvert Q\rvert, a, 1))$	$(12, a, 13, a, 1)$
h	$\#\#aaaa\#\quad N\#$	$(n, \#, h, N, 0)$	

$\mathfrak{Q}(x)$ Evolved to $\mathfrak{Q}(11010\ x)$

$(0, \#, 8, \#, 1)$ $(y, \#, h, Y, 0)$ $(n, \#, h, N, 0)$

$(x, \#, x, 0)$
$(x, a, t, 0)$

$(x, 0, v, \#, 0, (|Q| - 1, \#, n, \#, 1))$
$(x, 1, w, \#, 0, (|Q| - 1, \#, y, \#, 1))$
$(t, 0, w, a, 0, (|Q| - 1, \#, n, \#, 1))$
$(t, 1, w, a, 0, (|Q| - 1, \#, y, \#, 1))$

$(v, \#, n, \#, 1, (|Q| - 1, a, |Q|, a, 1))$
$(w, \#, y, \#, 1, (|Q| - 1, a, |Q|, a, 1))$
$(w, a, |Q|, a, 1, (|Q| - 1, a, |Q|, a, 1))$

$(|Q| - 1, a, x, a, 0)$
$(|Q| - 1, \#, x, \#, 0)$

$(8, \#, y, \#, 1)$ $(8, a, 9, a, 1)$
$(9, \#, y, \#, 1)$ $(9, a, 10, a, 1)$
$(10, \#, n, \#, 1)$ $(10, a, 11, a, 1)$
$(11, \#, y, \#, 1)$ $(11, a, 12, a, 1)$

$(12, \#, n, \#, 1)$ $(12, a, 13, a, 1)$

$\mathfrak{Q}(x)$ Evolving to Compute Some L_f

Each infinite downard path in the infinite binary tree corresponds to a unique language L_f, where string a^n lies in L_f if and only if the $n + 1$th branch of the downward path is a 1.

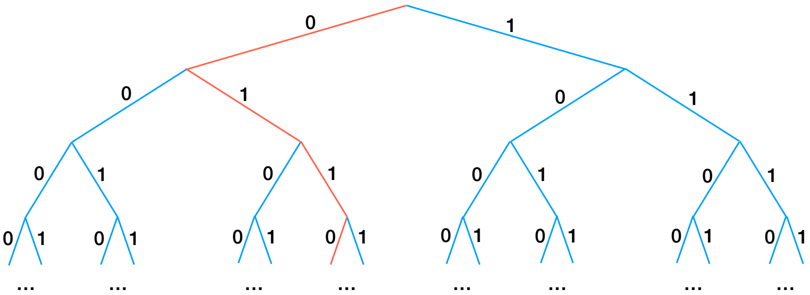


Figure: Infinite Binary Tree

$\mathfrak{Q}(x)$ Evolving to Compute Some L_f

An execution of $\mathfrak{Q}(x)$ on initial tape $\#\ \ \#a^n\#$ executes a random instruction $n+1$ times, creating a finite downward path of length $n+1$. After this execution, the descendant is $\mathfrak{Q}(f(0)f(1)\dots f(n)\ x)$, where $f(i)$ is the random bit measured in the $i+1$th execution of a random instruction.

LEMMA: Assume $i \leq n$. If $f(i) = 1$, then $\mathfrak{Q}(f(0)f(1)\dots f(n)\ x)$ on initial tape $\#\ \ \#a^i\#$ halts with tape $\#a^i\#\ \ Y\#$. If $f(i) = 0$, then $\mathfrak{Q}(f(0)f(1)\dots f(n)\ x)$ halts with $\#a^i\#\ \ N\#$.

THEOREM: For functions $f : \mathbb{N} \to \{0, 1\}$, the probability that language L_f is Turing incomputable has measure 1 in $(\nu, \mathfrak{L})$.

COROLLARY: $\mathfrak{Q}(x)$ evolves to compute a Turing Incomputable language L_f with probability measure 1 in $(\nu, \mathfrak{L})$.

$\mathfrak{Q}(x)$ Evolving to Compute $L_{h_{\epsilon_a}}$

Universal Turing machine / enumeration theorem, there is a Turing computable enumeration $\mathcal{E} : \mathbb{N} \rightarrow \{$ all Turing machines $\mathcal{M}\} \times \{$ Each of $\mathcal{M}$'s states as an initial state $\}$

This enumeration uses the blank-tape halting problem.

Set alphabet $\mathcal{A} = \{\#, 0, 1, a, A, B, M, N, S, X, Y\}$.

Let $\mathcal{M}_{\mathcal{E}}$ be the Turing machine that computes $\mathcal{E}_a : \mathbb{N} \rightarrow \mathcal{A}^* \times \mathbb{N}$, where tape $\#\ \ \#a^n\#$ represents the natural number n in the domain of $\mathcal{E}_a$.

$\mathbb{N}$ in the range of $\mathcal{E}_a$ holds the initial state of machine $\mathcal{E}_a(n)$.

$\mathfrak{Q}(x)$ Evolving to Compute Halting Language $L_{h_{\epsilon_a}}$

REMARK: For each $n \in \mathbb{N}$, with blank initial tape and initial state $\mathcal{E}_a(n)$ (2nd coordinate), then Turing machine $\mathcal{E}_a(n)$ (first coordinate) either halts or does not halt.

Define *halting function* $h_{\mathcal{E}_a} : \mathbb{N} \to \{0, 1\}$, where $h_{\mathcal{E}_a}(n) = 1$ if Turing machine $\mathcal{E}_a(n)$ halts with blank initial tape and initial state $\mathcal{E}_a(n)$. $h_{\mathcal{E}_a}(n) = 0$ if Turing machine $\mathcal{E}_a(n)$ does not halt.

Define *halting language* $L_{h_{\mathcal{E}_a}} = \{a^n : h_{\mathcal{E}_a}(n) = 1\}$.

THEOREM: The evolutionary path $\mathfrak{Q}(h_{\mathcal{E}_a}(0)\ \ x) \to \mathfrak{Q}(h_{\mathcal{E}_a}(0)\ h_{\mathcal{E}_a}(1)\ \ x) \to \dots\ \mathfrak{Q}(h_{\mathcal{E}_a}(0)\ h_{\mathcal{E}_a}(1) \dots h_{\mathcal{E}_a}(m)\ \ x)\ \dots$ computes halting language $L_{h_{\mathcal{E}_a}}$

Halting Complexity Questions

Can we find or define a measure of halting complexity to appropriately ask the next question?

(The Shannon complexity $|Q||A|$ is not adequate.)

Can this finite halting complexity of a Turing machine H be characterized as follows? There exists a threshold halting complexity $\theta(M)$ so that if M's halting complexity is greater than $\theta(M)$, then H cannot determine M's halting behavior.

(One approach is to assume an initially blank tape to assure that there is not complexity hidden in the different initial tapes.)

Self-Modification Questions

For a fixed Turing machine M, do there exist ex-machine self-modification procedures in some ex-machine X that start with a finite number of standard, random and meta instructions that can evolve to determine M's halting behavior with probability measure 1?

Does there exist a sufficiently high halting complexity for M, where these self-modification procedures fail?